Introduction to Linear Algebra & Commutative Algebra

Introduction

- • What is Linear Algebra?
- The branch of mathematics dealing with vector spaces, linear equations, and matrices.
- • What is Commutative Algebra?
- Study of commutative rings and their ideals, fundamental in algebraic geometry and number theory.
- • Why study both together?
- Both support understanding structures in algebra and abstract mathematical reasoning

Key Concepts in Linear Algebra

- • Vector Spaces & Subspaces

- • Linear Combinations and Span

- • Linear Independence

- • Basis and Dimension

- • Examples: $\mathbb{R}^n$, polynomials, matrices

Applications of Linear Algebra

- • Solving systems of linear equations
- • Matrix operations and transformations
- • Eigenvalues and eigenvectors
- • Real-world applications: computer graphics, machine learning, data science

Key Concepts in Commutative Algebra

- • Commutative Rings: Rings with ab = ba
- • Ideals: Special subsets of rings
- • Homomorphisms and Quotient Rings
- • Integral Domains, Fields, Units
- • Examples: $\mathbb{Z}$, polynomial rings $\mathbb{R}[x]$

Interconnection Between Both

- • Modules over rings (generalization of vector spaces)

- • Polynomial rings in linear algebra problems

- • Algebraic geometry using both: varieties as solutions of polynomial equations

- • Ring theory helps in understanding matrix structures

Sample Problems

- • Solve a linear system using Gaussian elimination

- • Prove an ideal is maximal in a ring

- • Identify a basis for a given vector space

- • Determine if a ring is a field

Conclusion

- • Linear Algebra and Commutative Algebra are foundational in abstract mathematics.

- • They help build algebraic reasoning and problem-solving skills.

- • These subjects pave the way to advanced topics like field theory, algebraic geometry, and coding theory.

- • Mastering both ensures strong mathematical maturity and research readiness.